Visualization of Mathematical Concepts through Prior Experiences

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**Abstract**

In this review, the concept of visualization as a connection between prior experiences and knowledge through the use of the *mind’s eye* is introduced. It would show that experiences are exclusive to every human being, as well as cognitive processes are particular to every individual, such that cultural relevance is indispensable to mathematical learning. Selected examples would illustrate how visualization connects the student’s previous experiences and knowledge to provide context for the student’s learning. Moreover, visualization would appear as our most flexible and strong cognitive tool. Indeed, it would be recognized not only as a student’s view of a picture but also as a mental ability to scrutinize his prior experiences and knowledge in order to create connections.

*Keywords*: visualization, out-school experiences, in-school experiences, prior experiences culture, connection, mind’s eye.

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**Introduction**

Visualization is not just images; it can be the relationship between a cognitive idea and a lived experience (see Fig. 1). According to Woolner (2004) some people experience higher order visualization than others to the *mind’s eye* (p.18). Furthermore, Lloyd (1995) in his article A *Human Review of Visualization in Human Cognition* states that “… *visualization includes both internal and external representations that can extend to all cognitive and affective outcomes*” (p. 45). In a like manner, Morrison, Robbins, and Rose (2008) suggest that students’ engagement to learn and achieve is shaped simultaneously between their homes and schools; with this in mind, it is clear that students’ learning activities are thoroughly linked to students’ social background (pp. 439-440). Moreover, D’Ambrosio (undated) declares that ethno-mathematics rests in the recognition that every cultural group produces unique conducts of dealing with reality, and classifies them into methods that would be developed, improved and transmitted from parents to their children (p. 3). Many educators agree that student success is closely engaged to home literacy and language practices (Morrison et al., 2008; Villegas and Lucas, 2002; Gay, 2000; Ladson-Billings, 1994; Cahnmann and Remillard, 2002; Hollie, 2001; Howard, 2001a, 2001b; Jimenez and Gertsen, 1999; Wortham and Contreras, 2002). For that reason, researchers recognize the inherent influence of socio-cultural factors in the student learning process through visualizations.

According to Masingila, Samson, and Kimani (2010), the in-school mathematics knowledge acquires power when the students connect it to their everyday experiences (p. 89*).* All people have the ability to build knowledge out of visualization processes (Lloyd, 1995, p. 46). Not to mention Masingila, Davidenko, and Prus-Wisniowska (1996), who suggest that in-school mathematical practices should be linked to those out-school experiences in order to enable students to create connections between their in-school learned methods and their out-school experiences (p. 177). These experiences allow the students to form ideas before they learn them in school (Driver, Guesne, and Tiberghien, 1985a, p. 2); therefore, they provide context to students to use their imagination to interpret the in-school concepts (Masingila et al., 2010, p. 91). In other words, visualization provides meaning to rigorous coursework during real-world problems application (Darling-Hammond, and Friedlander, 2008, p. 17). Equally important, Ensign (2003) explains how out-school mathematical experiences can be used by educators as examples during their lessons to exemplify problems with a context, giving the student *connections* (Masingila, 2010, p. 89) to solve the mathematical problem (p. 417). As a result, students were able to discuss other issues related to social justice and to mathematics. For instance, students were able to rationalize why certain stores have higher prices by not selling certain items individually.

Richards and Millers (2009) declare that experiences are exclusive to every human being as well as cognition processes are particular to every individual, but the perception of an experience is different for each other (pp. 25-26).Some students are able to construct a deeper understanding than others due to their unique life experiences which give them context to understand. Consequently, it is possible to infer that individuals experience visualization through a different perspective and the outcome knowledge varies by significance; such that it is not necessary to imply images (Woolner, 2004, p. 22). People need to use experience to make sense of the world around them (Moje, Ciechanowski, Kramer, Ellis, Carrillo, and Collazo, 2004, p. 41). In other words, visualization gives students the space to focus on the problem in order to clarify their understanding before any generalization can happen (Piggott and Woodham, 2008, pp. 27, 29). Visualization processes uniquely connect students’ prior experiences to cognitive development in order to improve students’ mathematical achievement.

*Figure 1*

(Martinez based on Lloyd, 1995; Piggott and Woodham, 2008)

**Cultural Relevance**

Moreover, according to Gay and Cole (1967), units of measurement are determined by cultural influence more than aspects of geometric and arithmetic practice; in fact, according to Masingila, et al. (1996), occasionally ethnic groups express distinctive mathematical procedures exclusively to their unique situation (p. 190). Prior knowledge and beliefs, resulting from cultural and personal experiences, give the children access to an interpretative process which is recognized as learning (Villegas and Lucas, 2002, p. 73). Students’ prior experiences or knowledge provides them with a unique confidence in learning mathematical processes (Morrison et al, 2008, p. 439). It is important for students to identify a visualization to generate comprehension and be able to use a meaningful language to express it (Piggott and Woodham, 2008, p. 27*).* Bishop (1988) states that mathematics is performed by every cultural group under six universal activities, counting, locating, measuring, designing, playing and explaining, and every culture derives its mathematical knowledge from continued and deliberated engaging (pp. 182-183). Finally, Masingila (1994), Masingila et al. (2010), Boaler (1999), Ensign (2003) and Bishop (1988) mention that production of out-school knowledge is linked to cultural variances and produced by students’ experiences.

***Social Environment***

Boaler (1999) states that the framework of situated perspective, that sustain a framework of constrains and affordances, is “*sufficiently broad and nuanced to inform the relationships formed between learners, their mathematical understanding and the environments in which they work*” (p. 260). This is not only because of their unique experiences, but also knowledge acquired as part of their cultural community (Masingila, 1994, pp. 430-431; Masingila 1996, p. 189). Furthermore, these perspectives propose that practices and behaviors of students in mathematical situations are neither exclusively mathematical, nor personal, but surface as elements of the interaction produced between students and their community along different environments (Boaler, 1999, p. 260). *Situated theories of learning* propose that students practice to behave within specific communities(Boaler, 1999, p. 264). Students’ social environment affects their mathematical perceptions in-and-out school situations.

**Out of School Experiences Awareness**

For instance, Masingila et al. (2010) conducted a study to find out how the sixth and eighth grade students from Kenya perceive the use of out-school mathematics in relation to the in-school concepts (p. 89) and Boaler (1999) performed research of UK students for a period of three years as they moved from grade nine to eleven, in two schools, to investigate the cognition development that students acquire in-school and their capability to use it during out-school situations (p. 261). The study done by Masingila et al. (2010) included 18 students from an urban area and 18 from a rural area; during the research, students kept a log of their everyday activities where they used mathematics for five days (p. 93). Table One provides the frequency of these activities reported by the students.

STUDENTS’ OUT-OF-SCHOOL MATHEMATICS PRACTICE.

TABLE 1

Total number of statements for six mathematical activities by respondent groups

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Math**  **Activities** | **All students**  **(n=36)** | **Female**  **(n=20)** | **Male**  **(n=16)** | **Std 6**  **(n=18)** | **Std 8**  **(n=18)** | **Urban**  **(n=24)** | **Rural**  **(n=12)** |
| **Counting**  **Measuring**  **Explaining**  **Playing**  **Locating**  **Designing** | 288  134  12  9  6  3 | 166  63  7  6  1  1 | 122  71  5  3  5  2 | 147  74  6  5  4  1 | 141  60  63  4  2  2 | 198  87  7  2  5  3 | 90  47  5  7  1  0 |

(Masingila et al., 2010, p. 97)

The authors realized that they found the six fundamental mathematical categories described by Bishop (1988). Therefore, they agree that the mathematics that students perceive during out-school situations can be classified into Bishop’s six universal activities (Masingila et al., 2010, p. 94). Articles from Masingila (1994) and Masingila et al. (1996) conducted studies where those universal activities were found (See Table of Five).

Table of Five

|  |  |  |
| --- | --- | --- |
| Author  &  Publication Date | Title | Key-Points |
| Masingila  et al.  2010 | Understanding students’ out-of-School Mathematics and Science Practice | \*Promote use of imagination, and critical thinking.  \*Experiences produce context to support connections.  \*The way students perceive their out-school mathematics in relation to their in-school concepts. |
| Morrison et al.  2008 | Operationalizing Cultural Relevant Pedagogy: A Synthesis of Classroom-Based Research. | \*Students’ learning is produced simultaneously between their home and school and it is related to socio-economic, cultural and language practices.  \*Prior experiences provide students confidence in learning mathematics |
| Woolner  2004 | Words or Pictures? Comparing a Visual and a Verbal Approach to Some Year 7 Mathematics. | \*Some people experience higher order visualization than others to the *mind’s eye*  \*Individuals experience visualization through a different perspective and the outcome knowledge varies by significance |
| Jo Boaler  1999 | Participation, Knowledge and Beliefs: A Community Perspective on Mathematics Learning | \*Investigate students’ ability to connect their in-and-out school experiences.  \*Teaching strategies: Traditional textbook and Mixed-ability  \*Production of out-school knowledge is linked to cultural variances and produced by experiences |
| Masingila  et al.  1996 | Mathematics Learning and Practice in and out of School: a Framework for Connecting these Experiences. | \*Out-and-in school mathematics are different but they complement each other.  Experiences provide value to in-school learning  \*Cognition and experiences are acquired as part of a cultural community |

(Martinez based on Masingila et al., 2010; Morrison et al., 2008; Woolner, 2004; Boaler, 1999; Masingila et al., 1996)

***Six universal mathematical activities***

Counting is the utilization of a methodical technique to evaluate and arrange distinct situations. It may involve calculations, or keeping track of a sequence, or the use of specific number names or words (Bishop, 1988, p. 182).

*Masingila et al. (2010) provide the following examples knowing the amount of change one should receive when buying things at a shop, calculating costs, figuring profit and loss, keeping track of the score in football, counting papers, counting mugs to be sure they are all there, counting while skipping rope, counting money to take for the bus, knowing how much one has saved, counting beats in music, keeping track of how many things one has sold in a shop, and keeping track of the distance one swims or one cycles.* (p. 95)

As Bishop (1988) sees it, “*Locating is exploring one’s spatial environment and conceptualizing and symbolizing that environment, with models, diagrams, drawings, words or other means*” (p. 182). Masingila et al. (2010) provide the next examples, “… *the goalkeeper seeing the angle from which the ball is coming from, marking points on a graph or map, and locating a song in a book by using a known song number*” (p. 95). According to Eisenhart (1988), human activity is inherently associated to human learning through individual experience such that a reproduction of these should be aligned according to them (p. 102). Most people have been given or have received instructions to get somewhere, such that they have to complement the verbal instructions with an imaginary map according to their previous experiences in the area.

Bishop (1988) explains that “*Measuring is quantifying qualities for the purpose of comparison and ordering, using objects or tokens as measuring devices with associated units or ‘measure-words’*” (pp. 182–183). For instance, choosing the right size cooking pot, the approximation of time for water to boil, the measurement of medication dose to be taken, the calculation of the time between notes in music, the approximate weight of meat after cooked, the measurement of time of travel to the super market, the calculation of time to be in line, the temperature set on a gas cooker (Masingila et al., 2010, pp. 95-96). Most of these activities are performed every day and without being noticed.

Designing is the creation of a figure or sketch for an item or for any segment of an individual’s spatial environment*.* Bishop (1988) states *“… it may involve making the object, as a ‘mental template’, or symbolizing it in some conventionalized way”* (p. 183). Masingila et al (2010) provided the following examples to draw a draft of a design one creates and describe how it works, to create a needlecraft, to build a car, and design storage bin (p. 96). This activity requires using imagination and several mathematical concepts.

Playing, according to Bishop (1988), “…*is devising, and engaging in, games and pastimes, with more or less formalized rules that all players abide by*” (p. 183). For instance, *“… guess my number, jumping game with three sticks, using the rules to find the weaknesses of the opposing team, and knowing how to play the game so it is fair”* (Masingila et al., 2010, p. 96). Participants are aware of the time between each play and add up points balances after each move to be aware of their position in order to the win the race.

Explaining is to discover ways to report the existence of phenomena; they can be spiritual, animistic or scientific (Bishop, 1988, p. 183). For instance,

*… being able to explain why the item you bought was the best buy, deriving something, explaining how to locate a song in a book by using a known song number, and using variables to represent quantities when explaining how things are related*  (Masingila et al., 2010, p. 96).

People do not realize that they are doing mathematics all the time. Table Two shows the percentage rate that participants reported being aware of using one of Bishop’s six mathematical activities.

TABLE 2

Percentages of respondents reporting six mathematical activities by respondent groups

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Math**  **Activities** | **All students**  **(n=36)**  **%** | **Female**  **(n=20)**  **%** | **Male**  **(n=16)**  **%** | **Std 6**  **(n=18)**  **%** | **Std 8**  **(n=18)**  **%** | **Urban**  **(n=24)**  **%** | **Rural**  **(n=12)**  **%** |
| **Counting**  **Measuring**  **Explaining**  **Playing**  **Locating**  **Designing** | 100  67  17  14  11  6 | 100  60  15  15  5  5 | 100  75  19  13  19  6 | 100  72  22  17  11  6 | 100  61  11  11  11  6 | 100  67  17  4  13  8 | 100  67  17  33  8  0 |

(Masingila et al., 2010, p. 97)

Pearson’s chi-square significance test says that the differences in Table Two could be statistically relevant, with a probability value of 0.02 (Masingila et al., 2010, p. 97). Activities that they identified as designing were not mentioned as often (Masingila et al., 2010, p. 97). Only two students reported these, both students lived in an urban area (Masingila et al., 2010, p. 97). Rural students did not recall designing activities (Masingila et al., 2010, p. 97). In addition, Table Two declares that rural students are more prompted to use mathematics through playing than urban students, which could be the result of less contact with technology. The high result of designing activity of the urban students can be result of the wide use of technology.

**Broad versus Narrow Viewers**

Masingila (2002), during a study in the USA of ten students in grade six and eight, defined as broader viewers to those students who include *" the idea that mathematics involves a way of thinking"*, and as narrower viewers to those students who identify mathematics as a subject that studies numbers and is learned in school (p. 36). As Masingila et al (2002) points out how broader viewers perceive themselves as using mathematics by *explaining, locating, and playing* versus narrow viewers who only reported classified activities as *counting, designing, and measuring* (Masingila, 2002, pp. 36-37). She concluded that three out of ten students were broader viewers—30%--and the rest were narrow viewers. Masingila et al (2010), who established the same rapport, classified only seven out of 36 students—19%--as broader viewers of mathematics, which is the minority of the participants.

**Traditional versus Mixed-ability Teaching Methods**

During Boaler´s (1999) study at Amber Hill School (AHS), mathematics was taught using a *traditional textbook* (Romberg and Carpenter, 1996) approach to approximately 200 students. In this case, teachers explained at the beginning of the lessons the methods and procedures from the chalk board; then the students practiced the procedures in textbook questions (Boaler, 1999, p. 262). Concurrently, at Phoenix Park School (PPS), mathematics was taught in Mixed-Ability groups to approximately 100 students. The Mixed- Ability group’s criterion supports educators to present several problems and ideas which students would be able to investigate and find a solution through different mathematical methods (Boaler, 1999, p. 262). Table Three describes the main points for each teaching method.

TABLE 3

SCHOOL APPROACHES

|  |  |
| --- | --- |
| Amber Hill School | Phoenix Park School |
| Textbooks  Short, closed questions  Teacher exposition every day  Individual work  Disciplined  High work rate  Followed a national curriculum  ‘Homogeneous’ ability groups | Projects  Open problems  Teacher exposition rare  Group discussions  Relaxed  Low work rate  Followed a national curriculum  ‘Heterogeneous’ mixed-ability groups |

(Boaler, 1999, p. 263)

***Students’ mathematical perception***

The approach used by AHS leads most of the students to believe that in-school mathematics is just about numbers and memorization of several rules, different formulas and equations (Boaler, 1999, p. 263). Consequently, many students believed that in-school mathematics did not require thought. These students were consistent with those that Masingila (2002) describes as narrower viewers (p. 36). None of the 12 students were able to understand that calculating the square units of yards or meters of carpet was equal to calculating the area of the same (Masingila et al., 1996, p. 185). Therefore, I conclude that by using traditional instruction methods, where the visualization between in-and-out of school mathematics is not supported, educators are limiting the spectrum of the students view, and the students are not being fully beneficiated by the education system.

In contrast, the approach used by PPH was more like out-school practice (p. 269). The students were relaxed and worked as they do in their everyday practices. According to Masingila et al. (1996) persons in out-school context find an allied in mathematical procedures instead of the enemy to defeat in the problem solving activities (p. 182). During the project development, students learned to select, adjust and apply mathematical methods such that the idea of mathematics as a thinking, flexible subject becomes natural to them in other words, students make sense out of mathematical definitions learned in-school once they connect them in out-school situations (Boaler, 1999, p. 264) just like an experience.

***Different outcomes***

Unfortunately, most of the students view mathematics intimately linked to arithmetic as used in school practices (Masingila et al, 2010, p. 98). The PPS students overcame AHS students’ mathematical achievement in the General Certificate of Secondary Education (GCSE) test that contains procedural and conceptual mathematics questions in a ratio of 2:1 approximately. The PPS’ students achieved notably higher grades. Specifically, 88 percent of the Phoenix Park (PPS) participants passed the examination versus 71 percent of the Amber Hill (AHS) students (Boaler, 1999, p. 264). According to Pope (1999), who leads an analysis of five successful students of high school, AHS spent much of their time practicing strategies that would guarantee success in class, but failed in producing cognitive knowledge due to the lack of connections between in-and-out school experiences. Boaler (1999) pointed out the enormous impact that different experiences had upon students’ mathematical perceptions and behavior (p. 263). Furthermore, “*Without everyday mathematical experiences, in school learning is solely for the sake of learning*” (Masingila et al., 1996, p. 177). At AHS the students learned conducts and practices appropriated to their classroom community. Lave (1993) insists that people use knowledge that depends on their activities, interests, goals and how these relate to the situation they are in to accept it.

*Perhaps for these students and for many students learning mathematics in formal school settings, what they perceive as mathematics is confined to academic mathematics (which is often perceived by elementary students to be arithmetic) and does not include a broader view of mathematics such as a way of thinking and making sense of data…we believe that it is important for researchers and teachers to work to support students in the construction of third space where their in-school and out-of-school mathematics and science practices may enable them to be more powerful in both practices* (Masingila et al, 2010, pp. 106, 108).

The Traditional Method had taught AHS students to expect more challenge in later exercises in a book so usually they would use more complicated procedures even if this was unsuitable. Furthermore, Schoenfeld (1985) asserts that this sort of cue based behavior is formed in response to conventional pedagogic school practices in mathematics that demonstrates specific routines that should be learned. In school situations, every exercise would be more difficult than the previous one. Since a student does not encounter this situation out-school they are unable to visualize a relation between this behavior which is stressing *cue based* conduct instead of flexibility caused by the out-school experiences. However, this behavior is significant for those students who are looking to find ways to embed in-school ideas in learning experiences that are engaging and effective. Experience is needed to use in-school mathematics effectively.

Masingila et al. (1996) concluded that even mathematics learning and practice in-school and out-school are different, they can complement each other (p. 191). The authors stated that classroom lessons must help students to create connections between their prior experiences and academic knowledge (Masingila et al, 2010, p. 108). According to Darling-Hammond, L. and Friedlander, D. (2008), Schools for Equity, located in the State of California, teach their students using M*ixed-Ability* (Boaler, 1999, p. 270)teaching methods achieving that 80 to 100 percent of their students continue to higher education and support students to participate in their communities and gain daily knowledge (p. 15). It’s clear that schools that already enforce this link, they’re very successful.

**Conclusion**

When students face situations in their classroom different from their expectations, they get confused, not because of their mathematical deficiencies, but because of the restrictions to which they are attuned (Boaler, 1999, p. 266). The students are unable to visualize any relation between their in-and-out-school mathematical experiences because a particular set of beliefs and practices that they learned as the appropriate way to behave in-school activities (Scribner and Cole, 1981). Visualization is an ability to call up images or experiences to the *mind’s eye* (Woolner, 2004, p. 18) as students’ necessity to make sense out of a situation. Moreover,

*Villegas and Lucas (2002) declared that learning is defined precisely as the process by which students generate meaning in response to new ideas and experiences they encounter in school. In this interpretive process, learners use their prior knowledge and beliefs to make sense out of the new input. To overlook the resources children bring to school is to deny them access to the knowledge construction process* (p. 73).

Students may be very good in applying mathematical concepts in their out-school experiences but because of the lack of ability to visualize they are unable to connect their prior experiences or knowledge to in-school learning. Moreover, according to Piggott and Woodham (2008) “… *visualizing is at the heart of problem solving itself*” (p. 30). Equally important is to recognize people’s natural ability to adapt almost any object into a resource to support their visualization strategies which act as timely prop to clarify an idea as it emerges. Consequently, all teachers are encouraged to implement visualization strategies to provide the student with a creative problem-solving process (Lloyd, 1996, p. 54). However, almost nothing is done to turn it into a reality then the link between in-school and out-school mathematics is one of the goals in the mathematics classroom instruction (Masingila et al., 2010, p. 90). In my experience as a substitute teacher, I have noticed that even when students recognize out-school mathematical activities, they don’t count them as mathematical applications because those activities are part of their lives and mathematics application is exclusive to in-school activities. Researchers agree that students require not only formal in-school lessons, but to figure out the mathematical activities involved in out-school experiences, so the individuals can make wiser decisions as consumers, and as part of the community.

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